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Analytical Predesign of a Pitch Controller for Rotational Speed Regulation in the Time and Frequency Domain of a Multi-Megawatt Wind Turbine Based on Competing Criteria

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Introduction

- Rotational speed Ω set-point tracking of variable-pitch wind turbines often requires the tuning of PI-controller

Controller & Plant

$$\hat{F}_{ctrl} = \frac{K_P\left(T_I s + 1\right)}{T_I s}$$

$$\hat{F}_{plant} = \frac{K_\infty}{\left(T_1 s + 1\right)}$$

Closed loop Dynamics

(Poles, Zeros)

Zero/pole cancellation

T_{1wish}

Pole-placement

ω_0, D, T_D

simulation

analytical

Criteria (disturbance reaction)

Time domain (step gust):

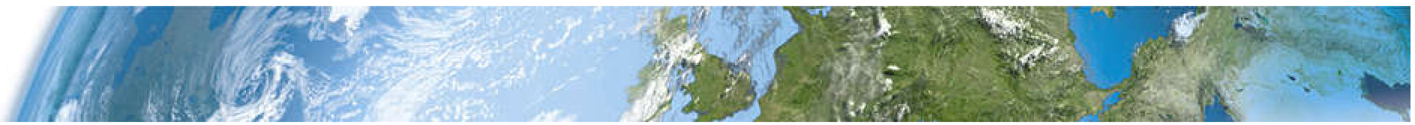
- Maximum actuator pitch rate $\dot{\Theta}_{max}$
- Maximum deviation $\Delta\ \Omega_{max}$ from a given set-point Ω_{com}

➔ Maxima in the time domain

Frequency domain (Turbulence):

- Maximum actuator pitch rate $\dot{\Theta}_{max}$
- Maximum deviation $\Delta\ \Omega_{max}$ from a given set-point Ω_{com}

➔ Maxima at a certain frequency, continuous loading



Introduction

- It is not about
  - a comparison of the two methods
  - the perfect PI-controller tuning
- It is about two methods for PI-controller tuning
  - One for extreme events like step gust in the time domain
  - One for continuous loading in the frequency domain
- **Objective:**
  - analytical advice
  - better physical understanding
  - good trade off
  - show what is possible and what is not possible
- Master objective: find easy to handle equations for a “paper and pencil” method usable for controller predesign

Outline

- Wind turbine model (linear)
- Disturbance models
- Closed-loop control loop
  - zero/pole cancellation
  - pole-placement
- Nonlinear simulation
- Summary

Wind turbine model

- Based on the law of conservation of angular momentum
- Transfer function: pitch $\delta\Theta$ ➔ rotational speed $\delta\Omega$

$$\hat{F}_{\Omega\Theta} = \frac{-\frac{\Omega_0}{P_0}\frac{\partial P}{\partial \Theta}}{\frac{I\Omega_0^2}{P_0}s+1} = \frac{K_{\infty\Theta}}{\left(T_1 s+1\right)}$$

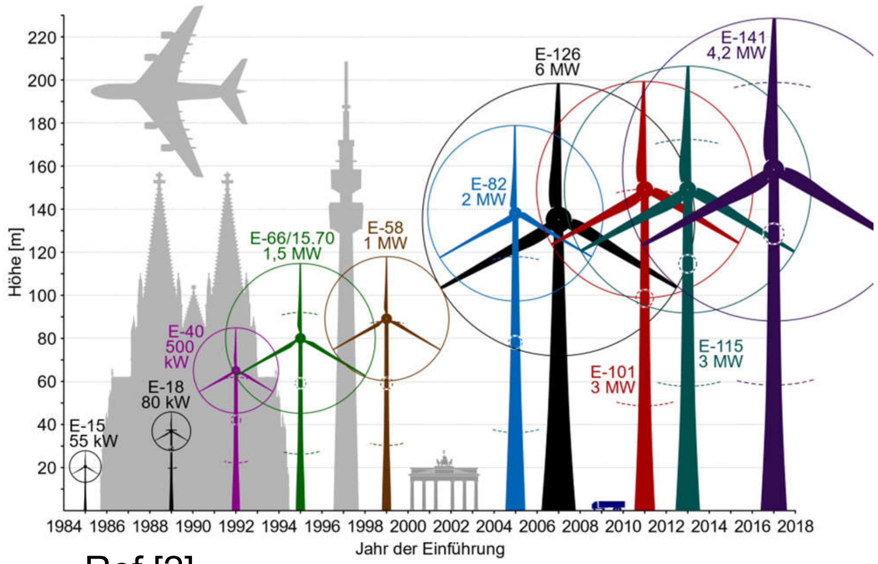
- Transfer function: wind δV_W ➔ rotational speed $\delta\Omega$

$$\hat{F}_{\Omega V_W} = \frac{-\frac{\Omega_0}{P_0}\frac{\partial P}{\partial V_W}}{\frac{I\Omega_0^2}{P_0}s+1} = \frac{K_{\infty V_W}}{\left(T_1 s+1\right)}$$

- Steady-state rotational speed $\Omega_0 = 12{,}1\ rpm$
- Steady-state power $P_0 = 5{,}3\ MW$
- Moment of inertia $I = 43{,}7 MKgm^2$
- Power-pitch sensitivity $\frac{\partial P}{\partial \Theta} = -25{,}0\frac{MW}{rad}$
- Power-wind sensitivity $\frac{\partial P}{\partial V_W} = -0{,}6\frac{MW}{m/s}$

Wind turbine model: NREL 5 MW

Description	Value
Rated Power (MW)	5.0
Rotor diameter (m)	126.0
Hub Height (m)	90.0
Rated Rotor speed (rpm)	12.1
Cut-in, Rated, Cut-out speed (m/s)	3.0, 11.4, 25,0



Ref [2]

NREL: National Renewable Energy Laboratory (Colorado, USA)

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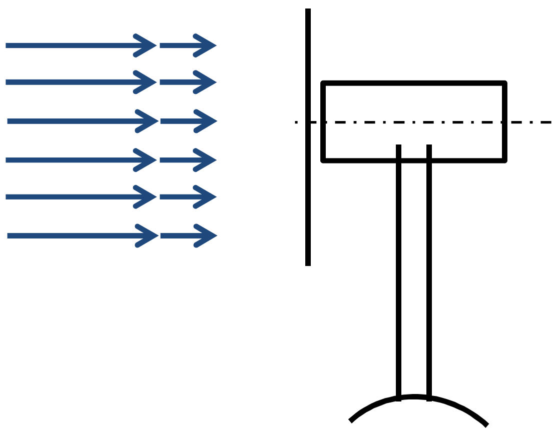
Disturbance models

- A step disturbance is a strong simplification of real gusts, since a step is not possible in nature
- First conservative (compared to a 1-cos gust) estimate
  - Max. disturbance reaction $\Delta\Omega_{\max}$
  - Max. actuator pitch rate $\dot{\Theta}_{\max}$
- Suitable for the rapid preliminary analytical design
- Considered: $\Delta V_W = 1\, m/s$ at $V_W = 12\, m/s$
- Linear system => scalable



Disturbance models

- 1-D approximation of real (measured) gust power spectral (PSD) density
- For simulation purpose zero mean Gaussian white noise is shaped through a Dryden form filter

$$\hat{F}_{V_W} = \underbrace{\frac{\sigma_{V_W}\sqrt{2T_{V_W}}}{1+T_{V_W}s}}_{Dryden}$$

standard deviation (turbulence strength): $\sigma_{V_W} = 1{,}8 \; m/s$

characteristic time constant: $T_{V_W} = 7{,}7 \; s$

wind speed: $V_W = 12 m/s$

- These values are tuned to meet the 3D Turbulence characteristics in the nonlinear simulation (FAST) at hub height
- low-pass characteristic ➔ high frequency components are attenuated ➔ acceptably frequency response approximation for analytical predesign

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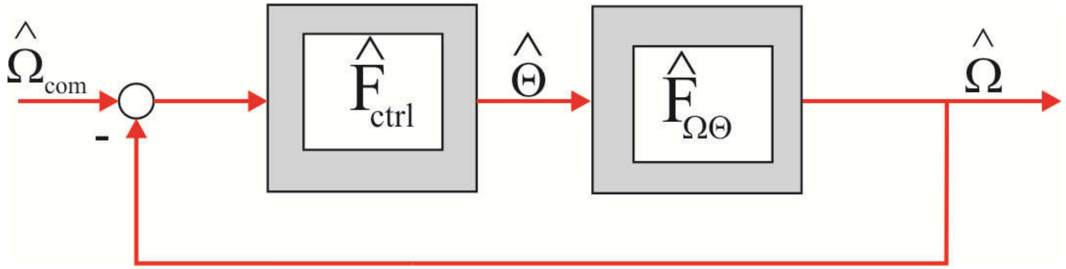
Closed-loop control: zero/pole cancellation reference reaction

PT$_1$

$$\hat{F}_o = \underbrace{\frac{K_P\left(\cancel{T_I s+1}\right)}{T_I s}}_{Controller}\underbrace{\frac{K_{\infty\Theta}}{\left(\cancel{T_1 s+1}\right)}}_{Plant} \Rightarrow \hat{F}_{\Omega\Omega_{com}\ FZ} = \frac{1}{\underbrace{\frac{T_1}{K_{\infty\Theta}K_P}}_{T_{1wish}} s+1}$$

$$\Rightarrow \begin{aligned} T_I &= T_1 \\ K_P &= \frac{T_1}{T_{1wish}K_{\infty\Theta}} \end{aligned}$$

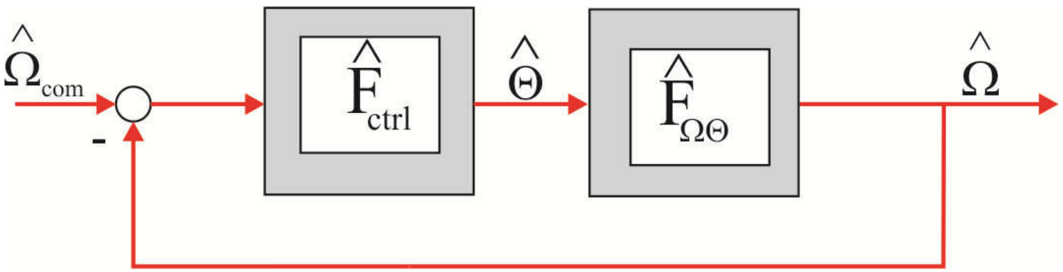


Closed-loop control: pole-placement reference reaction

PDT$_2$

$$\hat{F}_o = \underbrace{\frac{K_P\left(T_I s+1\right)}{T_I s}}_{Controller}\underbrace{\frac{K_{\infty\Theta}}{\left(T_1 s+1\right)}} _{Plant}\Rightarrow \hat{F}_{\Omega\Omega_{com}\ Pol} = \underbrace{\frac{\omega_0^2\left(T_D s+1\right)}{s^2+2D\omega_0 s+\omega_0^2}}_{reference\ system}$$

$$\Rightarrow K_P = \frac{2D\omega_0 T_1 - 1}{K_{\infty\Theta}}; \quad T_I = T_D = \frac{2D\omega_0 T_1 - 1}{\omega_0^2 T_1}$$



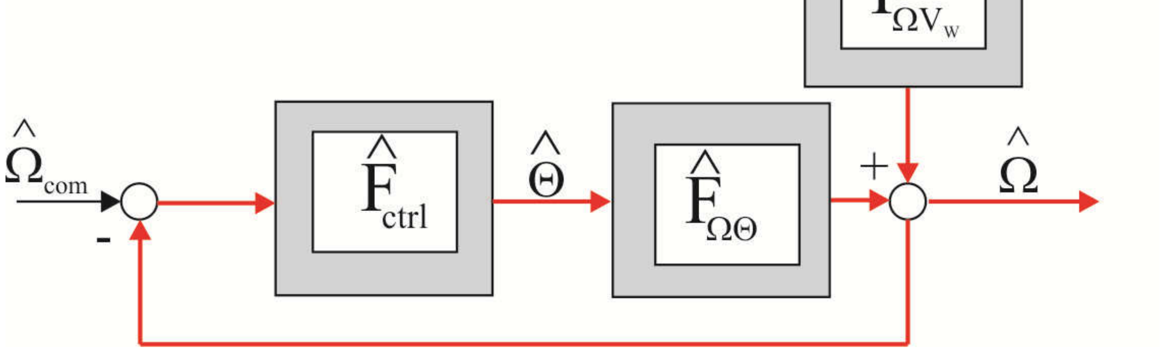
To prevent all-pass behavior (initial and steady state response a different) :

$$T_D = f\left(D,\omega_0,T_1\right) \geq 0 \Rightarrow \omega_{0\min} \geq \left(2DT_1\right)^{-1}$$

Closed-loop control: zero/pole cancellation disturbance reaction (time domain)

$$\hat{F}_{\Omega V_{WStep}\ PZ} = \frac{T_{1wish} K_{\infty V_W} s}{\left(T_1 s + 1\right)\left(T_{1wish} s + 1\right)}$$

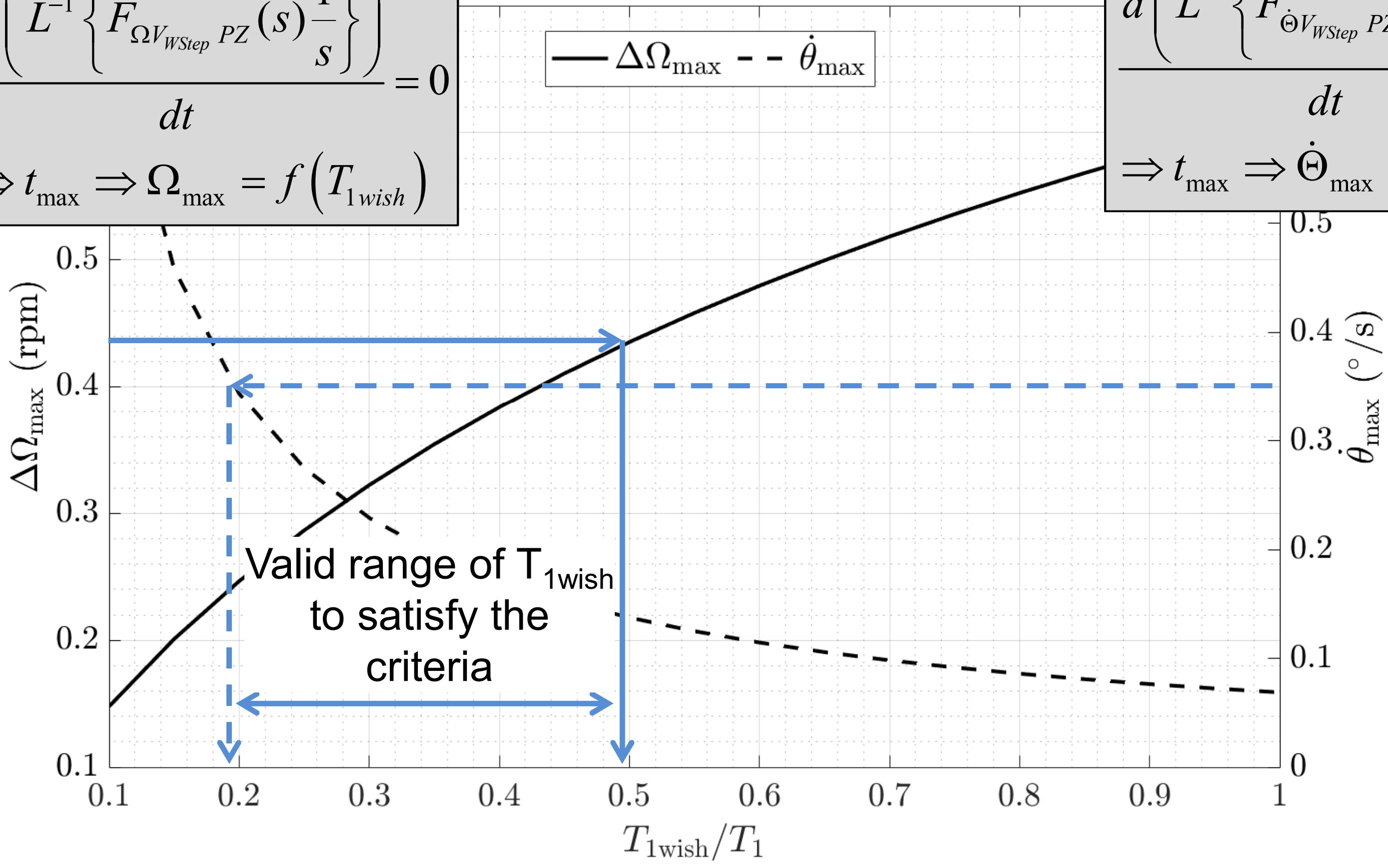
$$\hat{F}_{\dot{\Theta} V_{WStep}\ PZ} = \frac{K_{\infty V_W} s}{K_{\infty\Theta}\left(T_{1wish} s + 1\right)}$$



Closed-loop control: zero/pole cancellation disturbance reaction (time domain)

$$\frac{d\left(L^{-1}\left\{\hat{F}_{\Omega V_{WStep}\,PZ}(s)\frac{1}{s}\right\}\right)}{dt}=0$$
$$\Rightarrow t_{\max} \Rightarrow \Omega_{\max} = f\left(T_{1\,wish}\right)$$

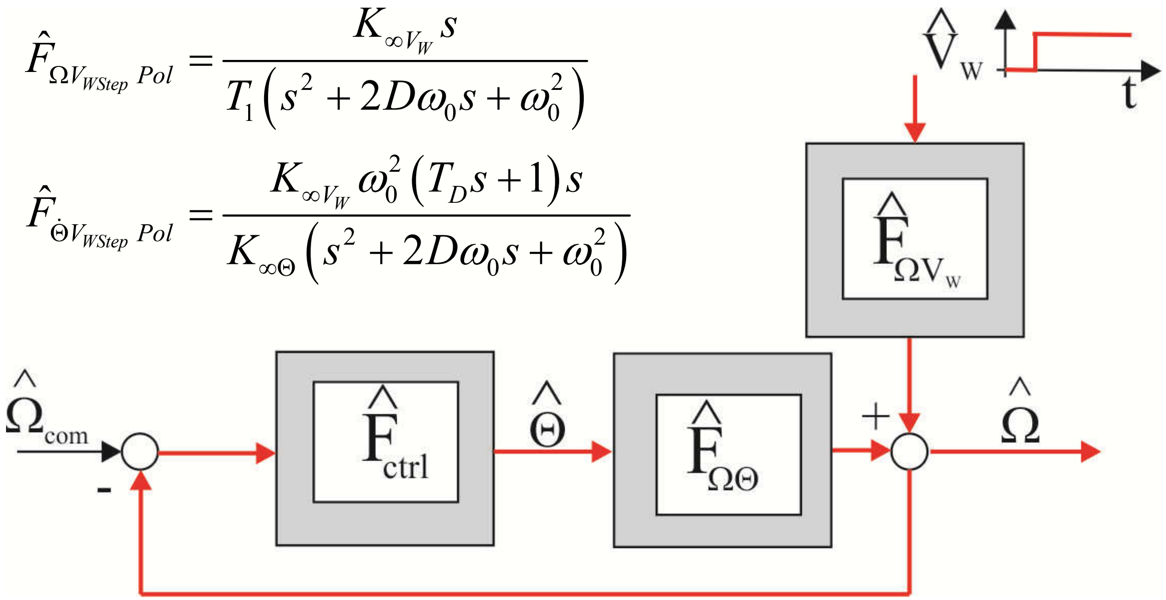
$$\frac{d\left(L^{-1}\left\{\hat{F}_{\dot{\Theta} V_{WStep}\,PZ}(s)\frac{1}{s}\right\}\right)}{dt}=0$$
$$\Rightarrow t_{\max} \Rightarrow \dot{\Theta}_{\max} = f\left(T_{1\,wish}\right)$$



Closed-loop control: pole-placement disturbance reaction (time domain)

$$\hat{F}_{\Omega V_{WStep}\ Pol} = \frac{K_{\infty V_W}\, s}{T_1\left(s^2 + 2D\omega_0 s + \omega_0^2\right)}$$

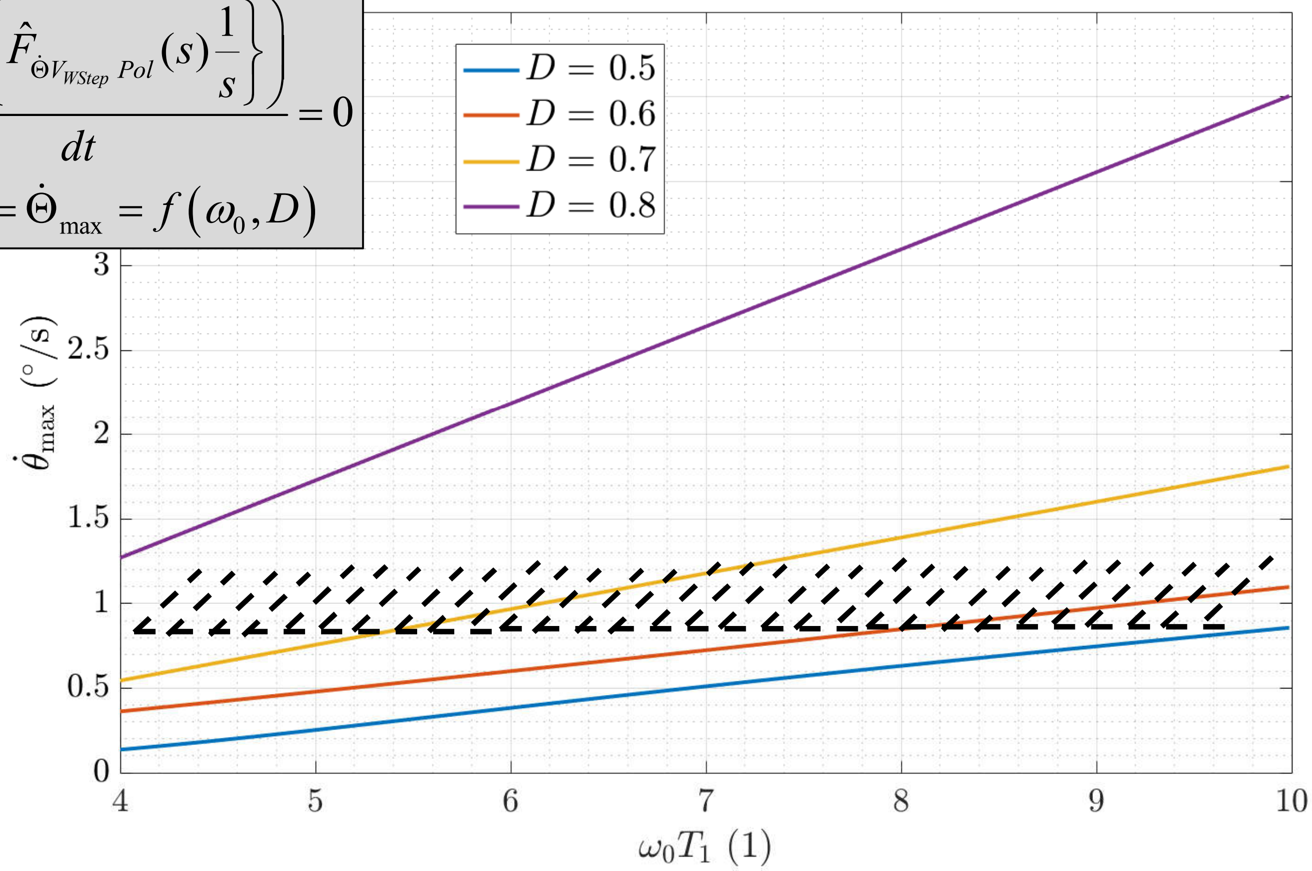
$$\hat{F}_{\dot{\Theta} V_{WStep}\ Pol} = \frac{K_{\infty V_W}\, \omega_0^2 \left(T_D s + 1\right) s}{K_{\infty\Theta}\left(s^2 + 2D\omega_0 s + \omega_0^2\right)}$$



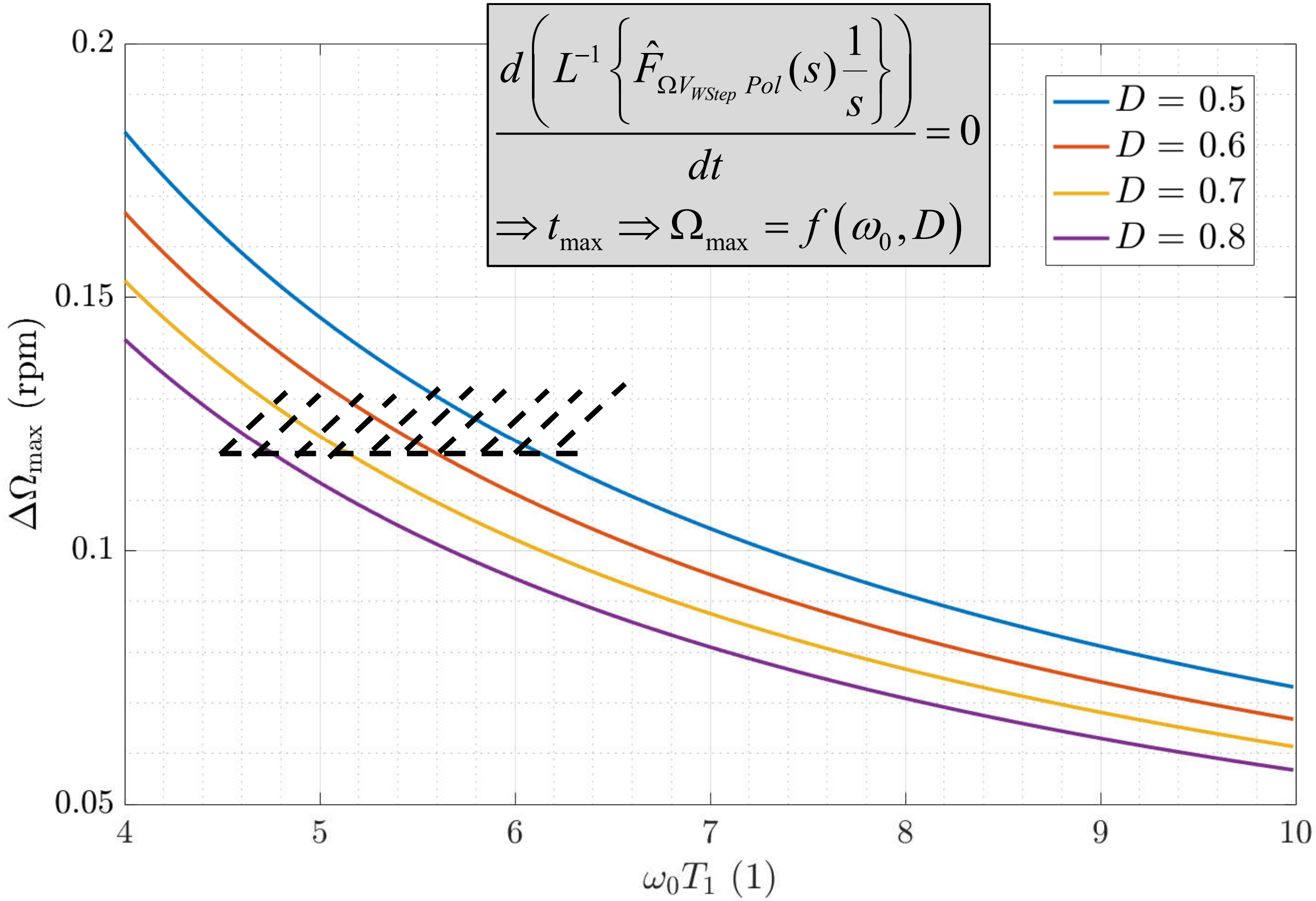
Closed-loop control: pole-placement disturbance reaction (time domain)

$$\frac{d\left(L^{-1}\left\{\hat{F}_{\dot{\Theta}V_{WStep}\,Pol}(s)\frac{1}{s}\right\}\right)}{dt}=0$$

$$\Rightarrow t_{\max} = \dot{\Theta}_{\max} = f\left(\omega_0, D\right)$$



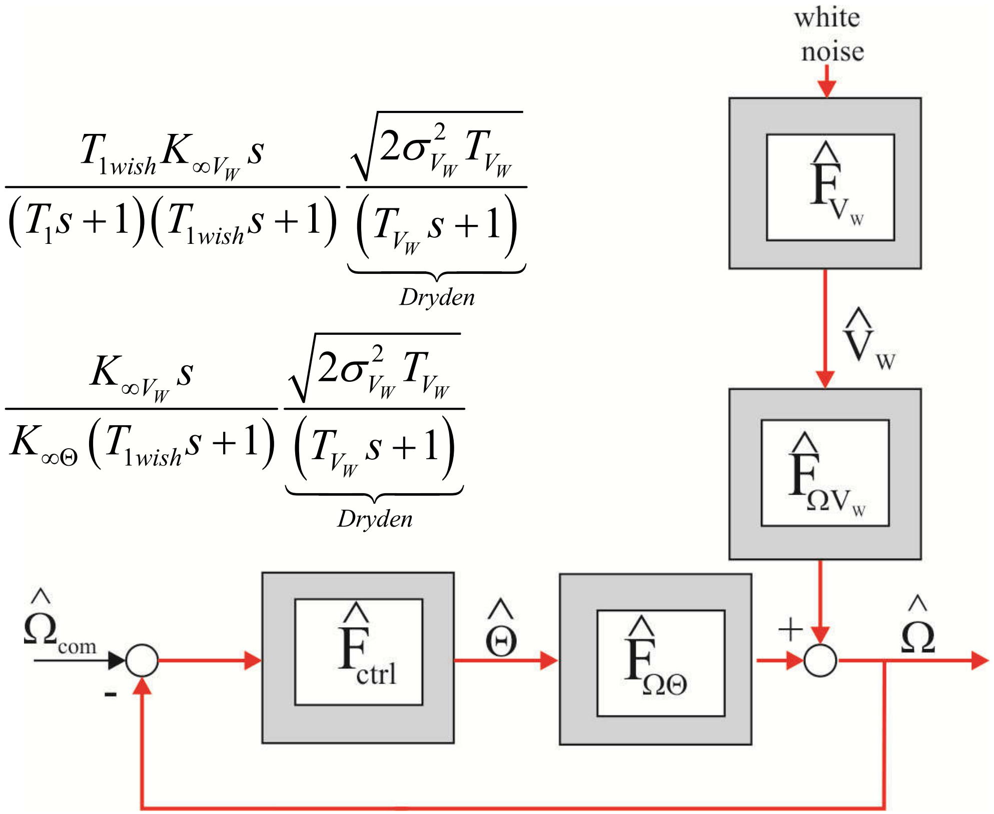
Closed-loop control: pole-placement disturbance reaction (time domain)



Closed-loop control: zero/pole cancellation disturbance reaction (frequency domain)

$$\hat{F}_{\Omega V_{WDry}\,PZ} = \frac{T_{1wish} K_{\infty V_W}\, s}{\left(T_1 s + 1\right)\left(T_{1wish} s + 1\right)} \underbrace{\frac{\sqrt{2\sigma_{V_W}^2 T_{V_W}}}{\left(T_{V_W} s + 1\right)}}_{Dryden}$$

$$\hat{F}_{\dot{\Theta} V_{WDry}\,PZ} = \frac{K_{\infty V_W}\, s}{K_{\infty\Theta}\left(T_{1wish} s + 1\right)} \underbrace{\frac{\sqrt{2\sigma_{V_W}^2 T_{V_W}}}{\left(T_{V_W} s + 1\right)}}_{Dryden}$$



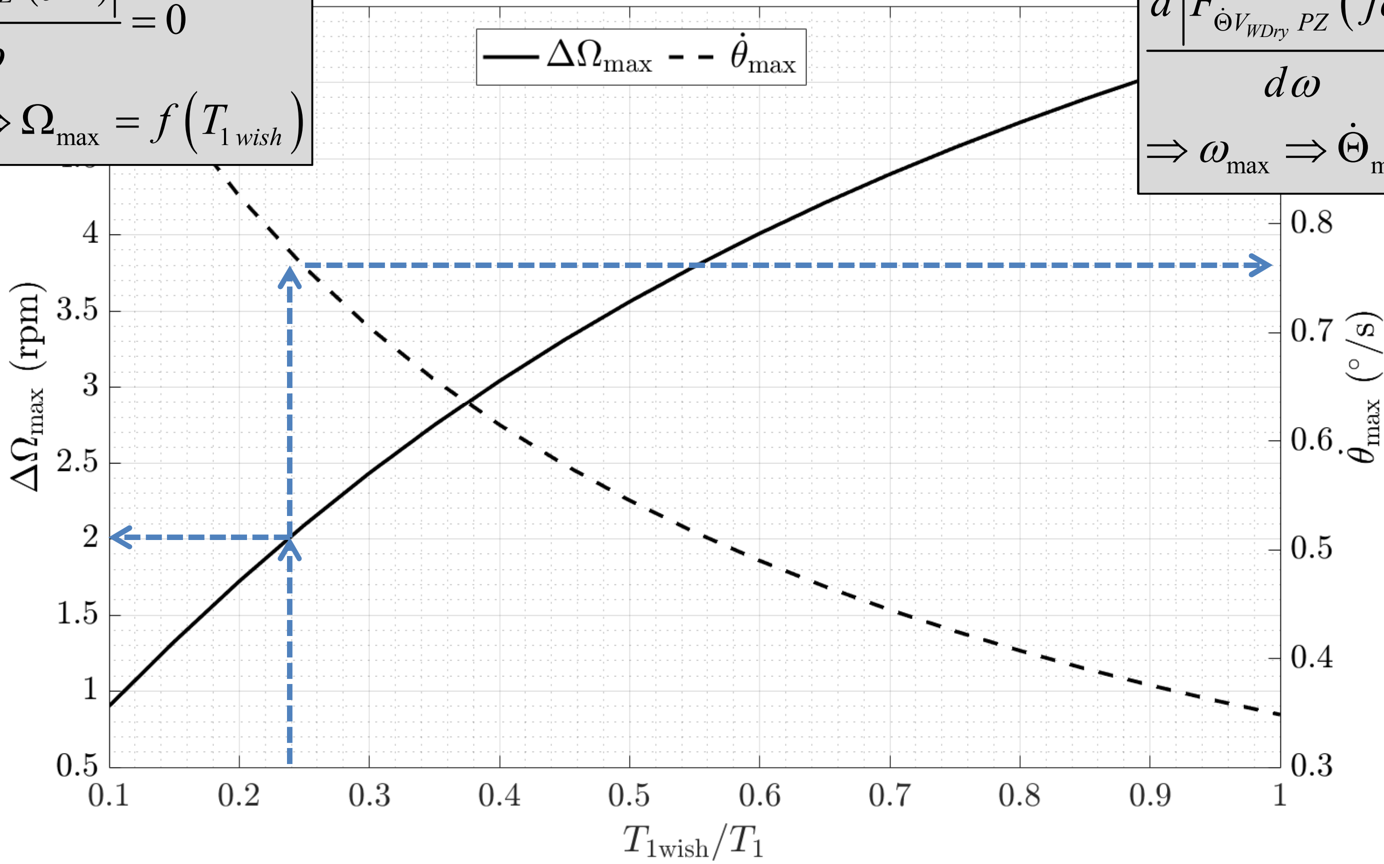
Closed-loop control: zero/pole cancellation disturbance reaction (frequency domain)

$$\frac{d\left|\hat{F}_{\Omega V_{WDry}\,PZ}\left(j\omega\right)\right|}{d\omega}=0$$

$$\Rightarrow \omega_{\max} \Rightarrow \Omega_{\max} = f\left(T_{1\,wish}\right)$$

$$\frac{d\left|\hat{F}_{\dot{\Theta} V_{WDry}\,PZ}\left(j\omega\right)\right|}{d\omega}=0$$

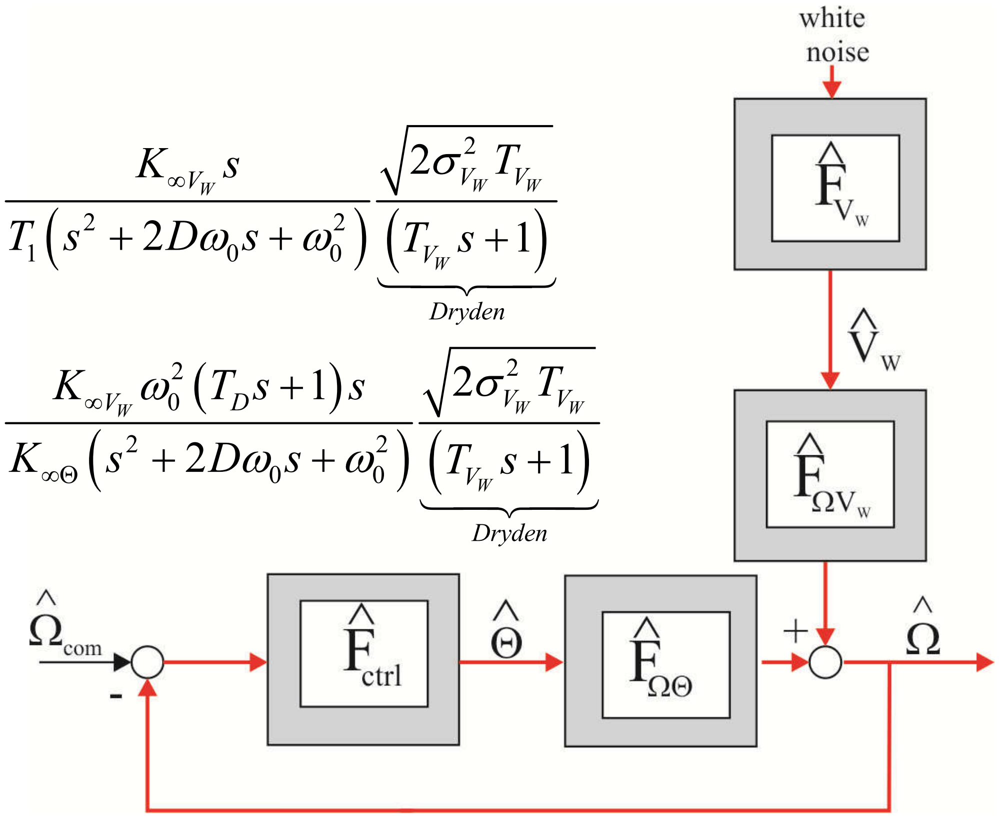
$$\Rightarrow \omega_{\max} \Rightarrow \dot{\Theta}_{\max} = f\left(T_{1\,wish}\right)$$



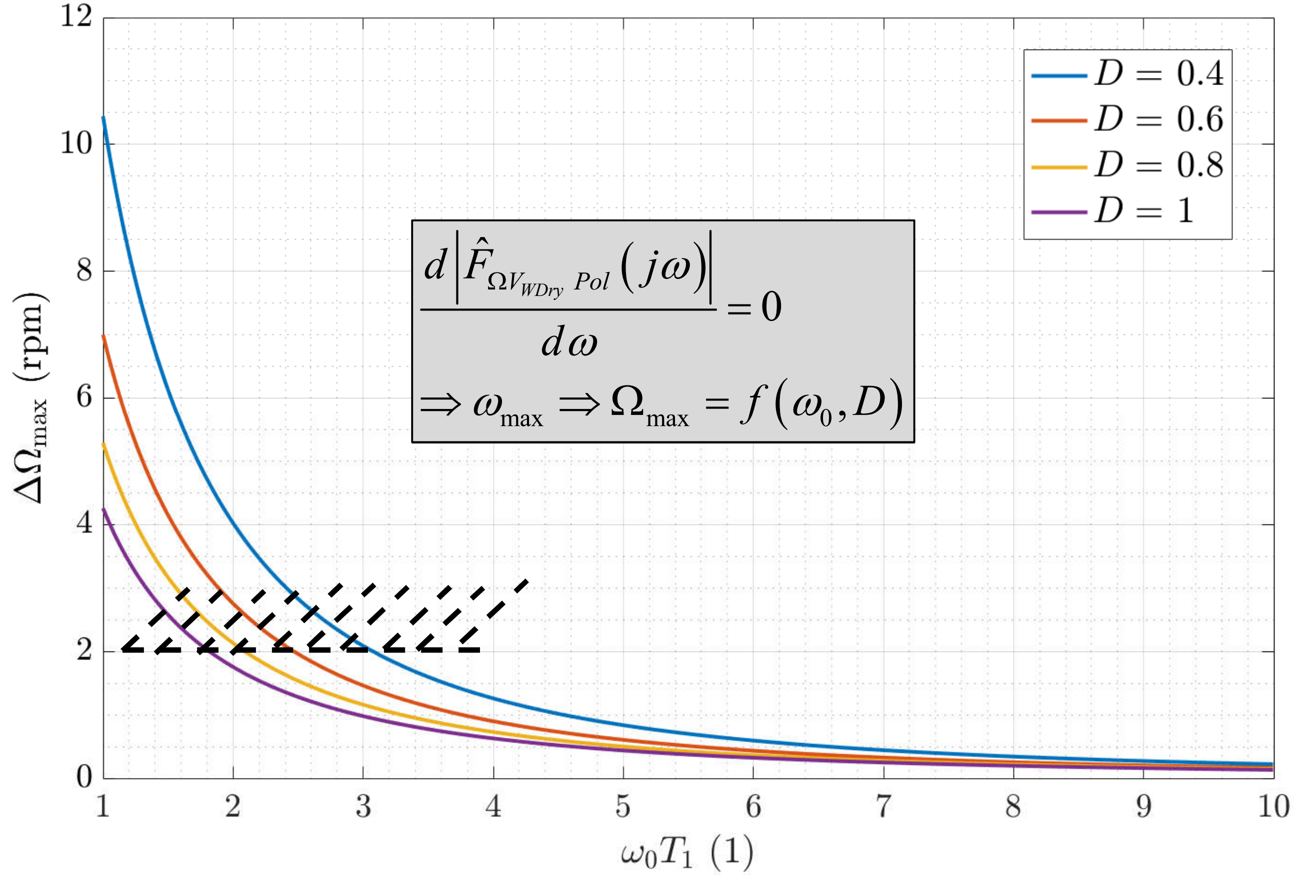
Closed-loop control: pole-placement disturbance reaction (frequency domain)

$$\hat{F}_{\Omega V_{WDry}\ Pol} = \frac{K_{\infty V_W} s}{T_1\left(s^2 + 2D\omega_0 s + \omega_0^2\right)} \underbrace{\frac{\sqrt{2\sigma_{V_W}^2 T_{V_W}}}{\left(T_{V_W} s + 1\right)}}_{Dryden}$$

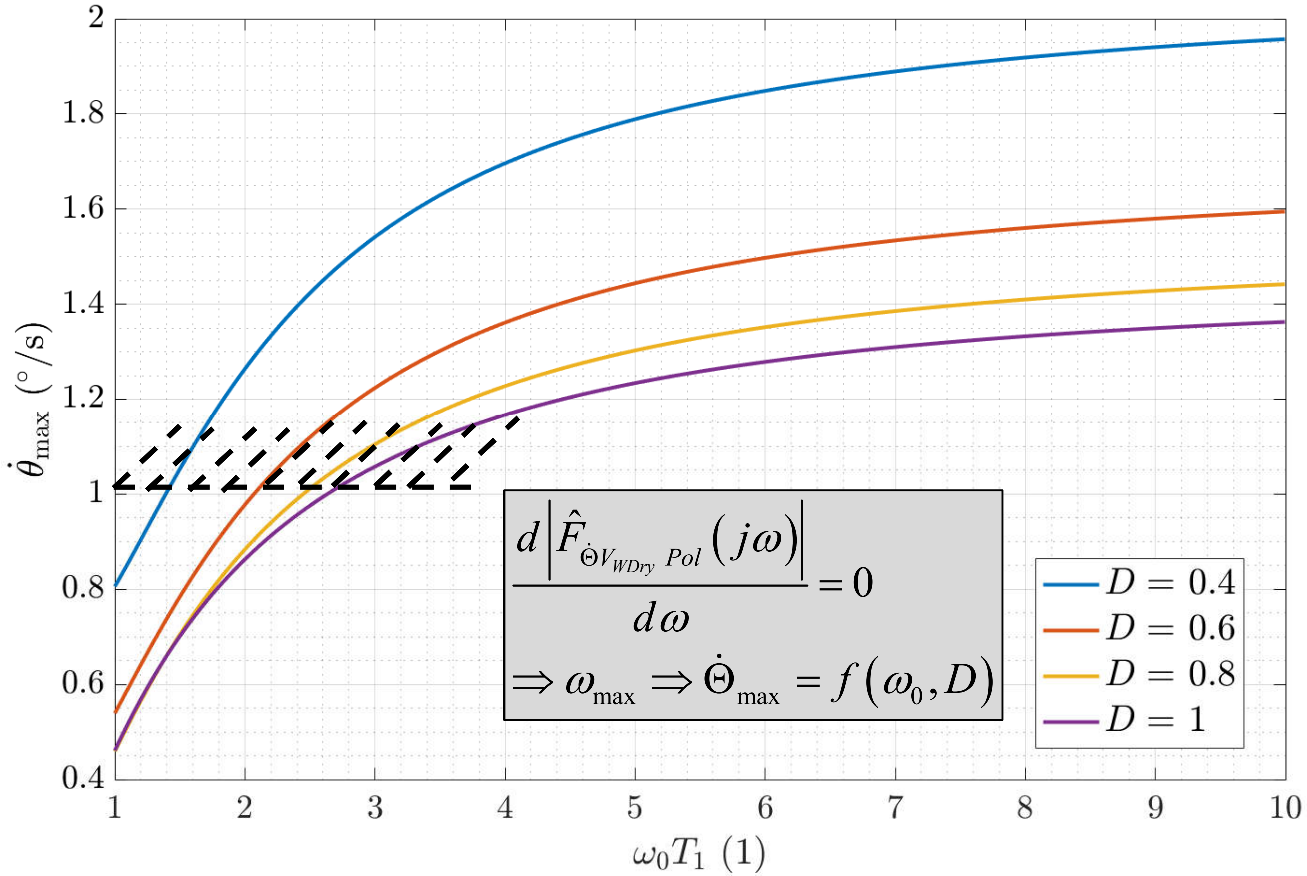
$$\hat{F}_{\dot{\Theta} V_{WDry}\ Pol} = \frac{K_{\infty V_W} \omega_0^2 \left(T_D s + 1\right) s}{K_{\infty\Theta}\left(s^2 + 2D\omega_0 s + \omega_0^2\right)} \underbrace{\frac{\sqrt{2\sigma_{V_W}^2 T_{V_W}}}{\left(T_{V_W} s + 1\right)}}_{Dryden}$$



Closed-loop control: pole-placement disturbance reaction (frequency domain)



Closed-loop control: pole-placement disturbance reaction (frequency domain)



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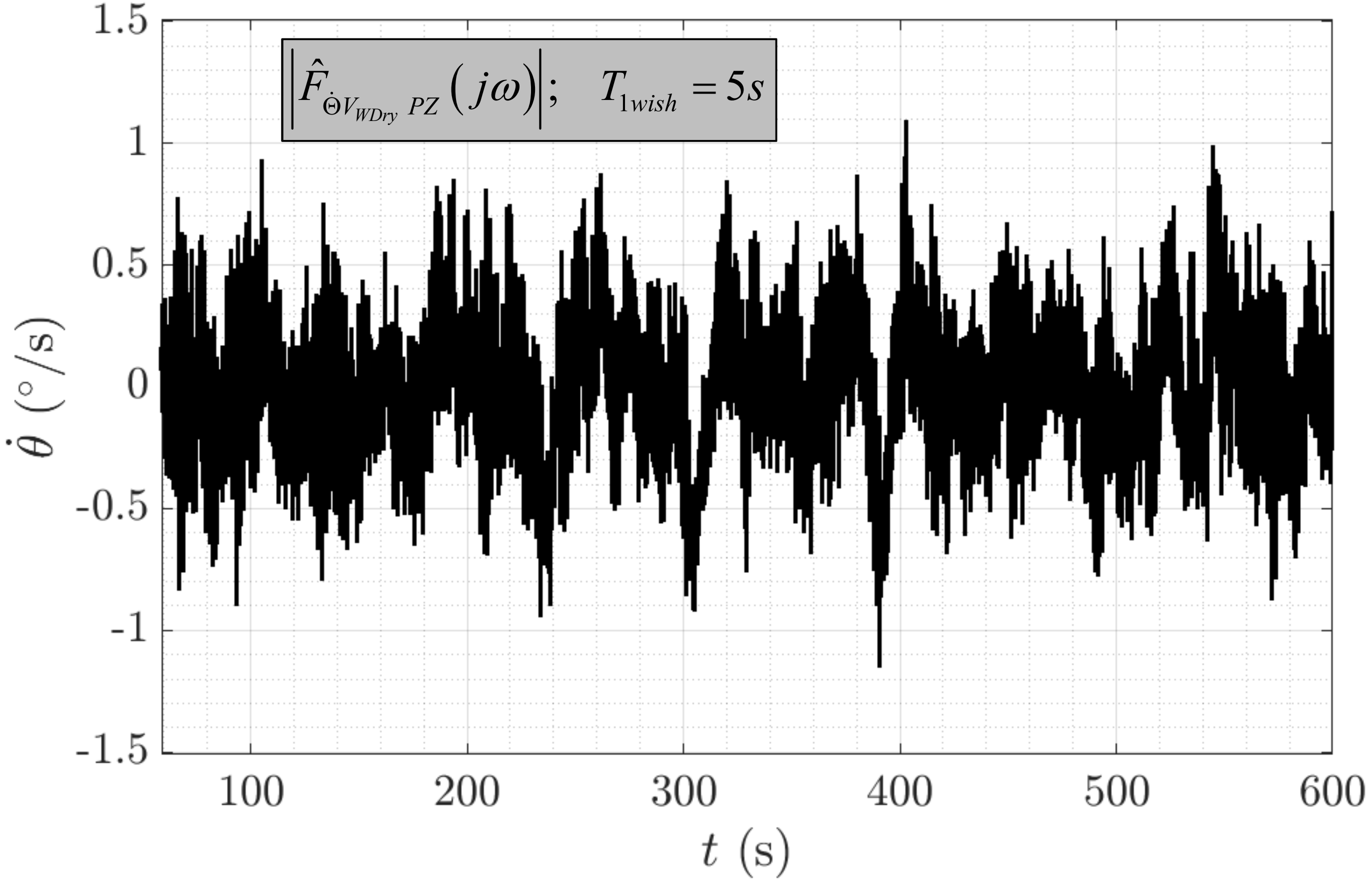
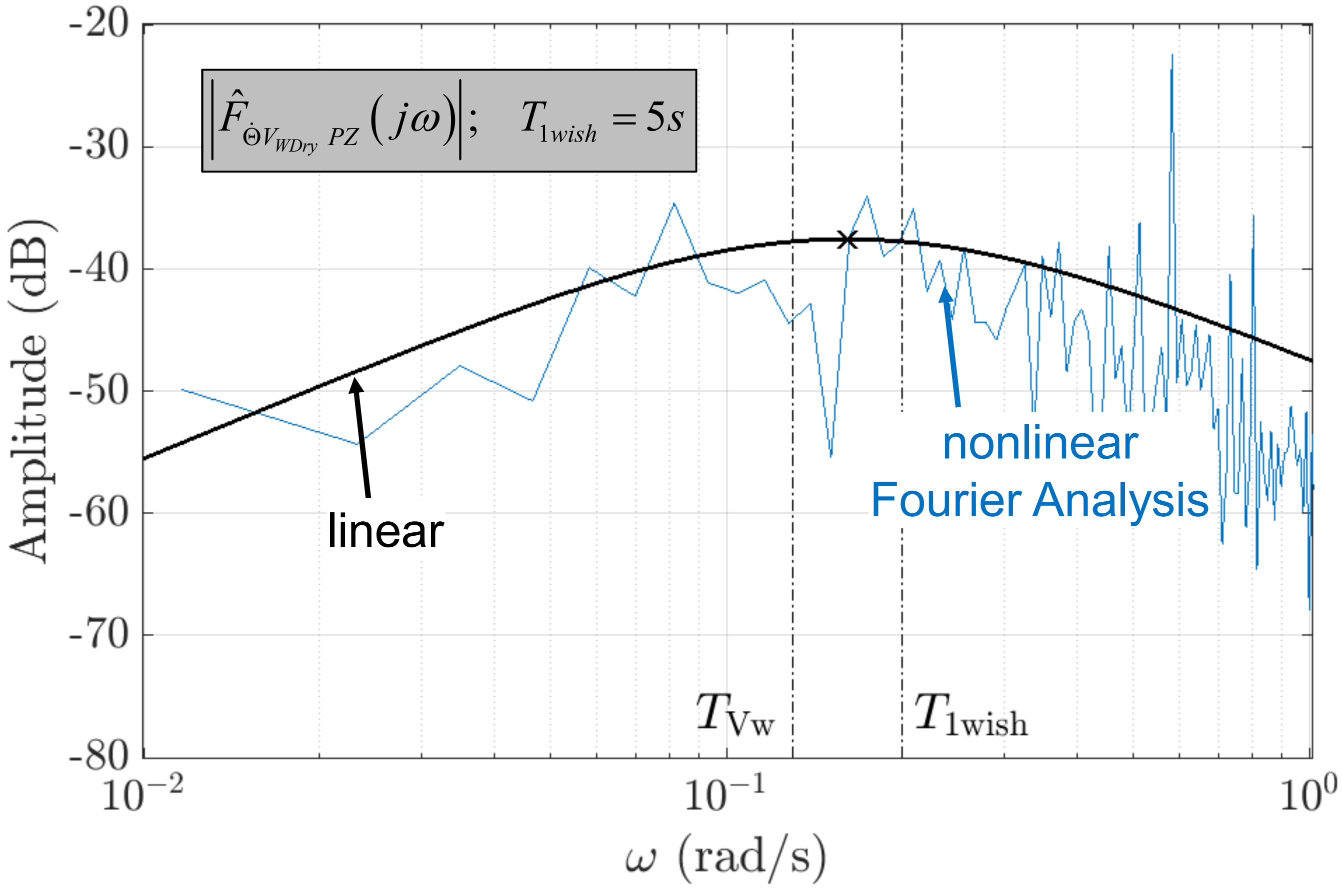
Nonlinear simulation (FAST)

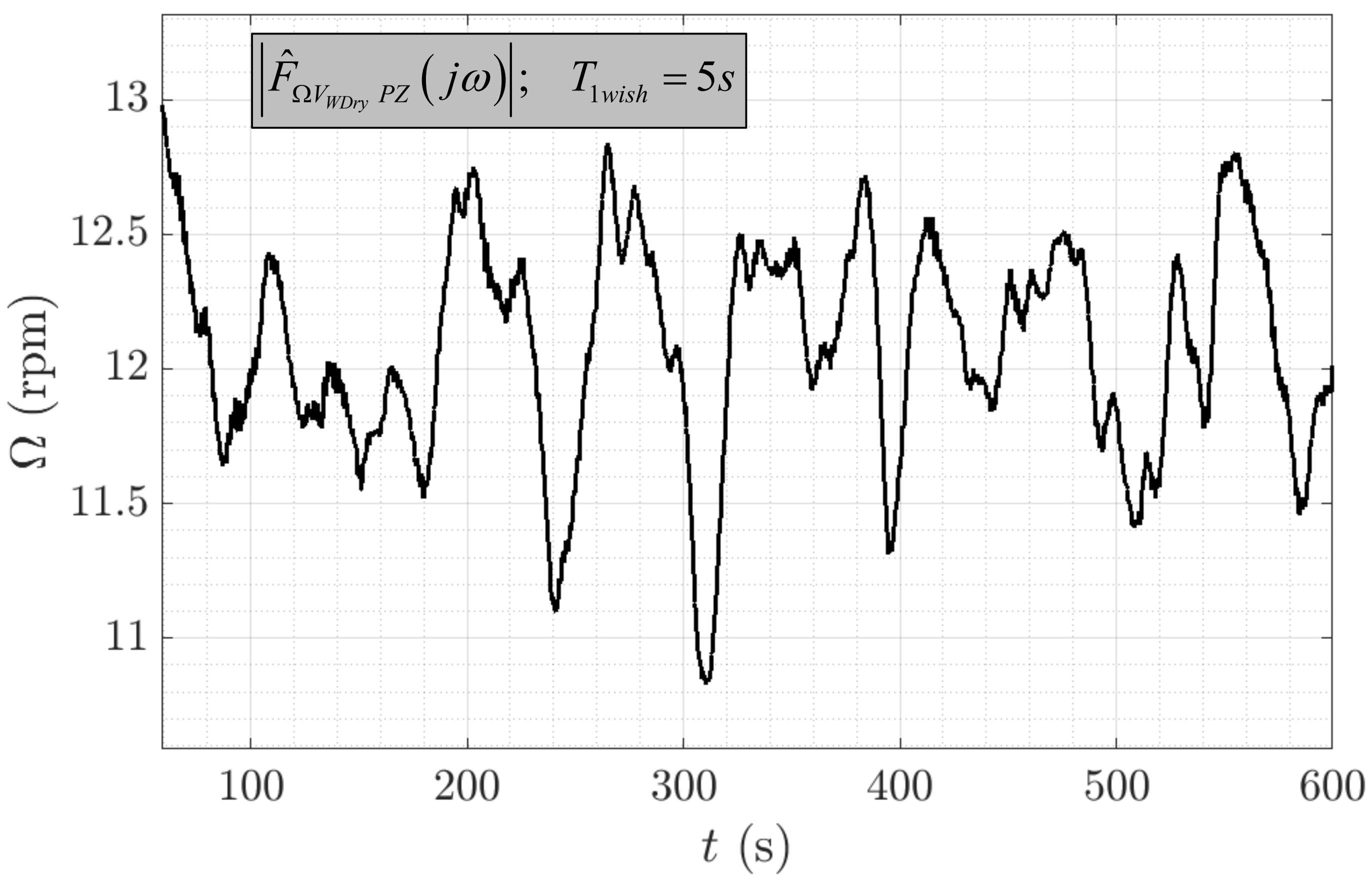
zero/pole cancellation disturbance reaction (frequency domain)



Nonlinear simulation (FAST)
zero/pole cancellation disturbance reaction (frequency domain)

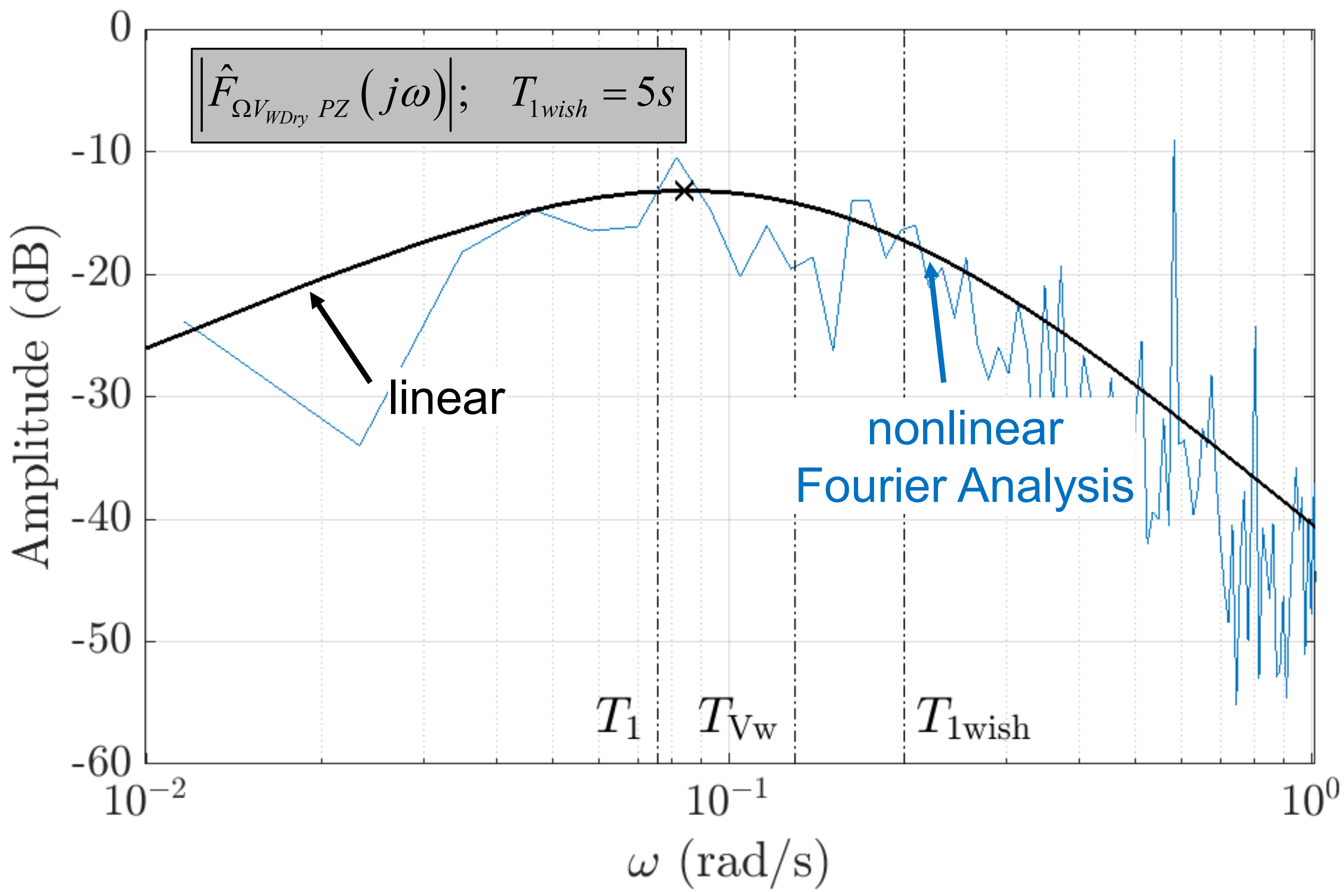


Nonlinear simulation (FAST)
zero/pole cancellation disturbance reaction (frequency domain)



Nonlinear simulation (FAST)

zero/pole cancellation disturbance reaction (frequency domain)



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Summary

- 2 analytical tuning methods for rotational speed Ω set-point tracking of variable-pitch wind turbines were shown
- Discussed by two criteria in the time- (step gust) and frequency-domain (1-D Dryden turbulence spectrum)
- Time and frequency domain results are hard to compare
- Give analytical advice to find a good trade off
- Deviate requirements for actuators
- Simple task (analytical tuning of a PI controller for a first order System) results in ambitious equations ➔ further discussion of the equations and simplifications for a rapid “paper and pencil” controller predesign
- Equations for additional criteria like robustness criteria (phase margin, …)

Thank you for your attention!

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